High Efficiency CT with Optimized Recursions (HECTOR^{TM}) [1]

Wolfram R. Jarisch

Cyber Technology, Inc. Potomac, MD 20854

www.cyber-inc.us

[1] Patent Pending

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Introduction

Background: Computed tomography (CT) is one of the important tools in micro-analysis to obtain whole sample 3-D structural density information. It is often used with a variety of complementary analysis methods because of its relatively non-destructive nature. Two major methods are used today to obtain this 3-D information: Filtered Back-Projection (FBP) and Simultaneous Iterative Reconstruction Technique (SIRT) [1]. Both have specific shortcomings: FBP requires a large number of projections, typically in the hundreds, while SIRT requires fewer projections and allows for density constraints; however, it requires massive computation and converges slowly - if at all.

Objective: The goal of the present effort is to develop a bridging technique that combines the efficient and rapid computations of FBP with the ability to use fewer projections and density constraints. An earlier effort representing such a bridge is High Efficiency Computed Tomography (HECT^{TM}) [2]: by choice of parameters it converges either into FBP or SIRT. Residual analysis, that is the difference between the actual and the (object-estimated) predicted projections, showed the potential for significant increases in speed: residual amplitudes persisting over iterations at the same location might be compensated through locally increased loop-gain, while oscillating residual values might be compensated by decreasing the local loop-gain. In essence, the new approach is to use a *matrix-based relaxation system* applied to residuals ("update-errors") instead of using a single relaxation factor to weigh corrective updates, such as in SIRT.

The Technical Problem

The Key Issues central to the application of the matrix-based relaxation system are:

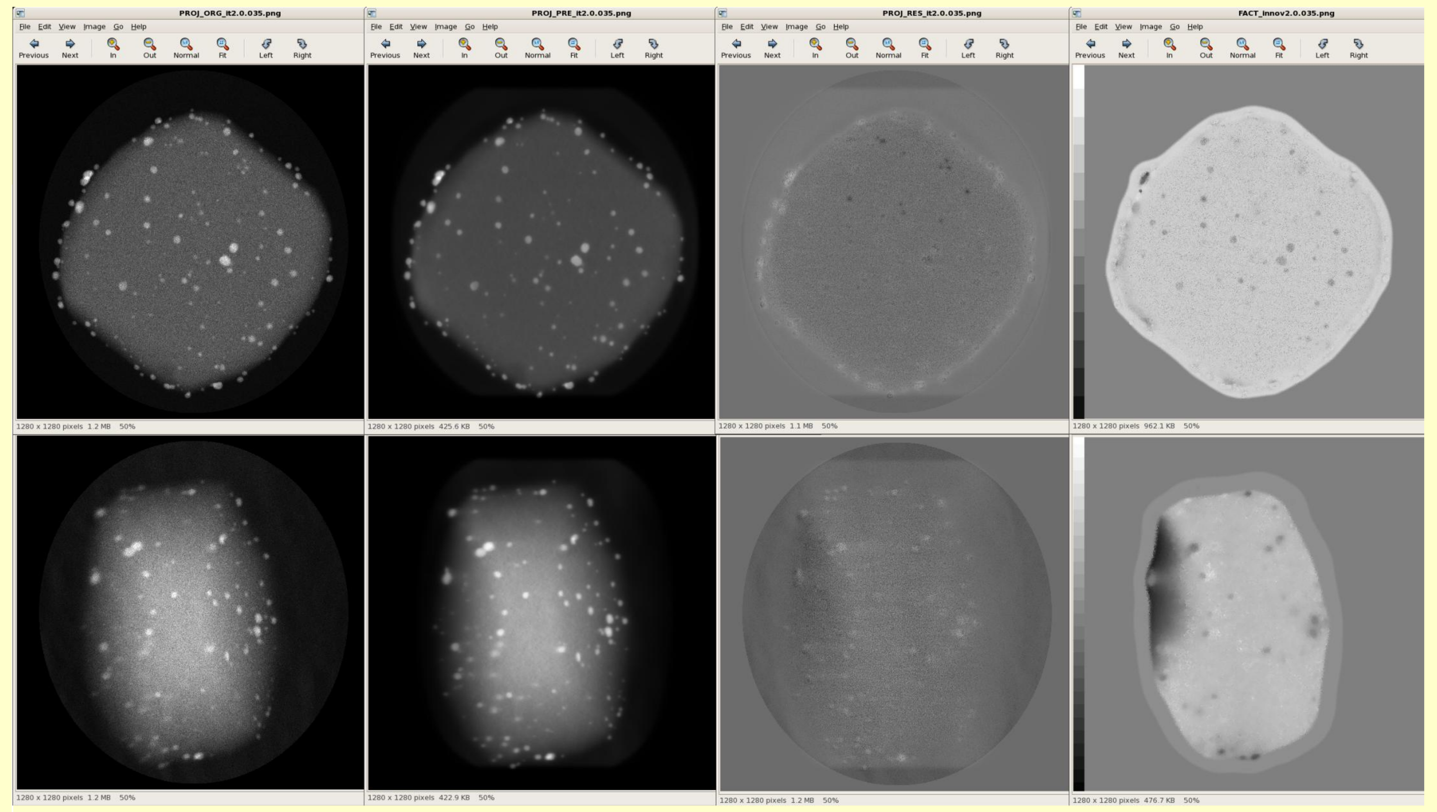
1. HECT^{TM} must operate in an approximately linear systems-response update region
2. Direct computation of the necessary gain / relaxation factors is not feasible due to the size of the matrices involved
3. The estimation of the gain matrices must be performed at every iteration
4. Additional reference items such as past residuals must be kept in memory

The Solution

The Technical Solution to these issues is:

1. Sufficiently limiting the update steps, if needed to make the linearizations valid approximations
2. Using a multigrid approach that supports valid linearizations without frequent step limitations
3. Iteratively analyzing the behavior of the innovation values and adjusting the gain / relaxation factors accordingly
4. Keeping a compressed representation of the history of residuals (a few bits/residual)

Result for Parallel Beam Geometry HAADF-STEM Imaging [6]



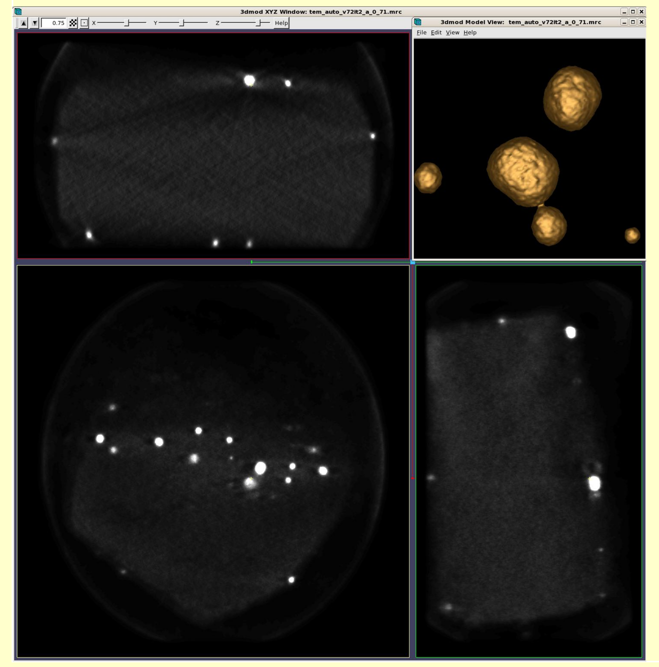


Fig. 1, Top: Two near-orthogonal views of $ErSi_2$ catalyst. From left to right: (a) original projection; (b) re-projection of estimated object density; (c) residual error; and (d) optimizing gain / relaxation matrix. Object density was estimated from 71 projections using 2 fine-grid iterations with HECTOR^{TM}.

Fig. 2, Left: Sample IMOD [5] renditions of 3-D reconstruction.

Result for Cone Beam Geometry X-ray Ultra Microscope [7]

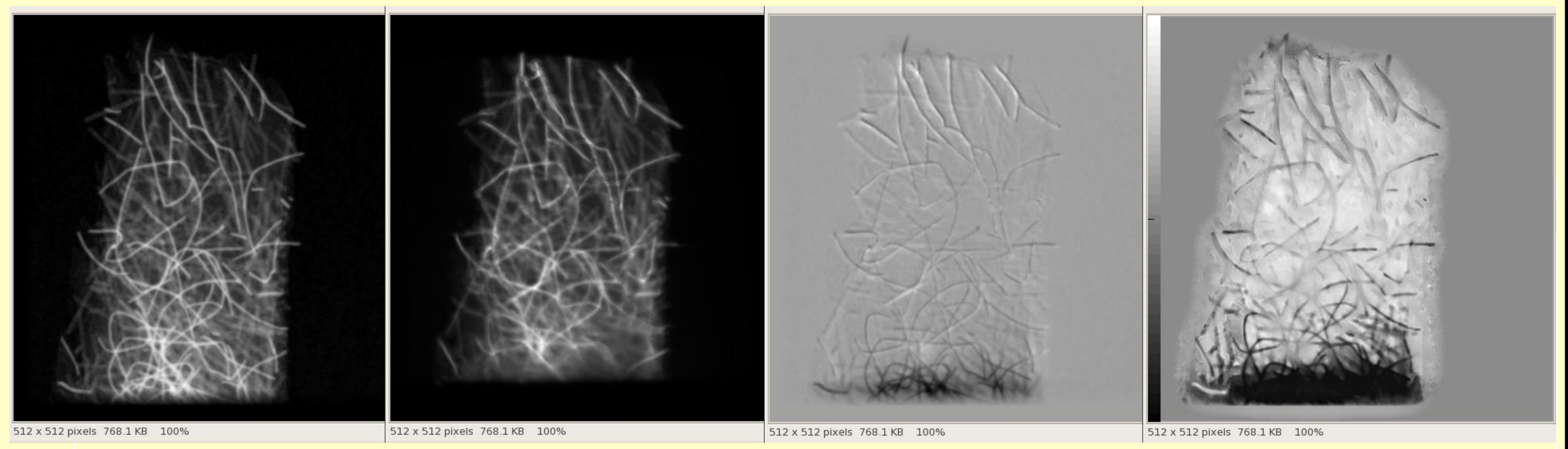
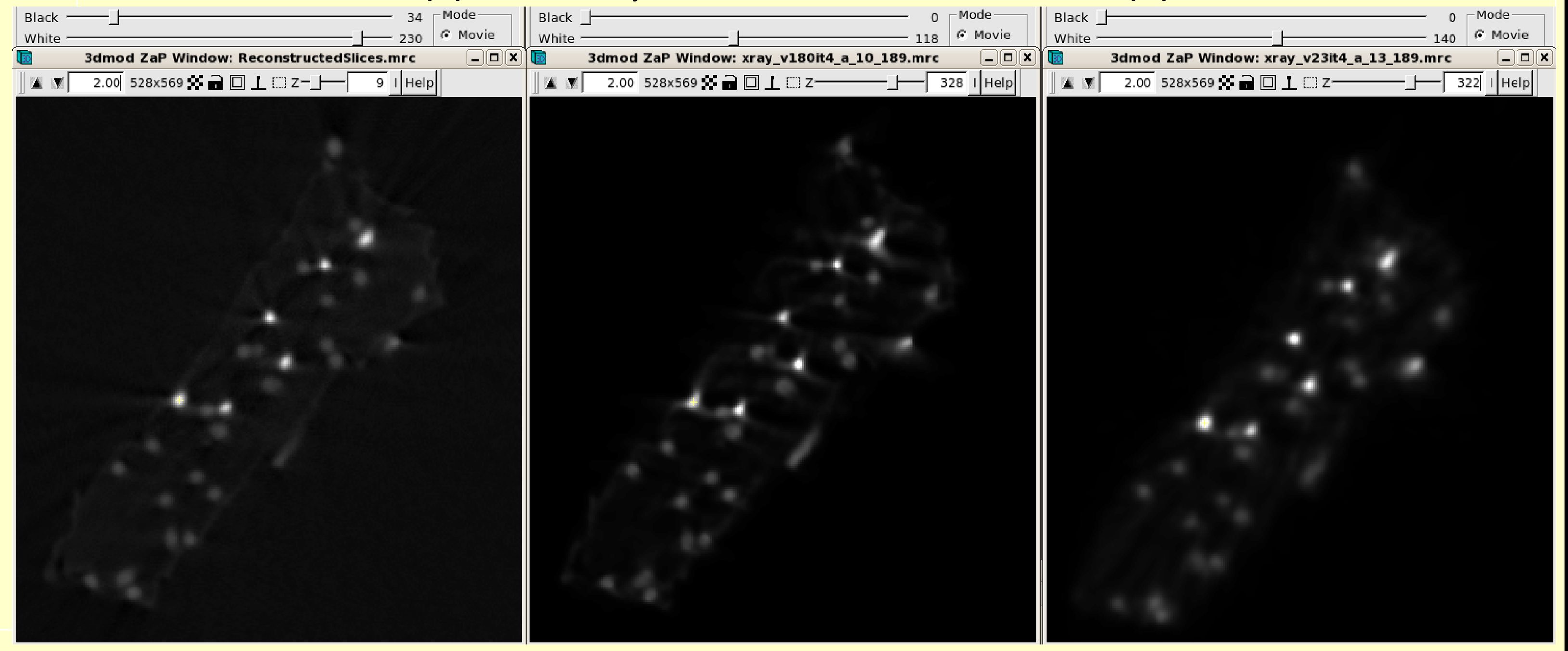


Fig. 3, Top: Wire mesh sample , from left to right: (a) original sample projection; (b) predicted projection from estimated object density; (c) residual error (difference between original projection and estimate of projection); (d) estimated gain matrix (1 of 180 views). Each gray box in (d) represents a 10% change in gain.

Fig. 4, Below: Sample IMOD slice renditions of the 3D reconstruction of the wire sample. From left to right: (a) 190-view FBP; (b) 180-view HECTOR^{TM}; (c) 23-view HECTOR^{TM}. Note that (c) uses only about 12% of the data of (a).



Discussion

Bayes Estimation is the gold-standard in statistical estimation. In linear Gaussian estimation problems the Kalman filter [3] provides the optimal "state" estimate. Here, the "state" represents the object density [4]. In practice, however, a number of approximations are needed to make computations feasible. In particular, the present approach requires a constrained solution for the "state" that is approximated by an extended Kalman filter. Actual matrix operations are in part replaced by spectral operations and the Kalman gain is estimated by a sensitivity analysis throughout the iterative multi-grid estimation: in particular when model residuals persist the gain values are increased and they are reduced when they alternate (see resulting sample gain matrices in Figs. 1d & 3d).

Summary

The new object density estimation technique structurally resembles the extended Kalman filter [3] and solves three key goals of present high resolution, high precision CT: speed, accuracy, and constraints. Implementing the new approach typically accelerates reconstruction by a factor of about p / 2, where p is the number of projections used. Many applications that require tens or hundreds of projections can be accelerated 10 to 100 fold by HECTOR^{TM}. Convergence is typically reached with 2 – 6 iterations at the highest grid resolution.

References

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[6] The author is grateful to Christian Kuebel of Karlsruhe Institute of Technology (KIT) for providing the HAADF-STEM catalyst data.
[7] The author is grateful to S. Fahey and C. Booth for providing projection data from Gatan's X-ray ultra Microscope (XuM).