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A Logarithmic Transformation of Projection Data and Material Density Representation derived from Probability Theory

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BACKGROUND

Efforts to reduce patient x-ray dose in tomography has led to non-linear algorithms. The main drawback has been the large number of iterations required to minimize the cost function. These time consuming iterations result from a poor characterization of the relationship among many variables that are used to characterize object density.

In addition to the above-noted shortcomings, the reconstruction becomes increasingly more demanding as the number of projections is reduced. In this case current algorithm's linearizations tend to become less efficient.

An alternate formulation of estimation leads to a logarithmic transformation of the projection data: consider two independent variables, say x and y with joint probability

$$p(x, y) = p(x) * p(y) \qquad (1)$$

Taking logarithms on both sides [for simplicity assume p(.) > 0] yields addition operations instead

$$\log[p(x, y)] = \log[p(x)] + \log[p(y)] \qquad (2)$$

familiar from back-projection. Changing notations with P(.) = log[p(.)] yields equivalently

$$P(x, y) = P(x) + P(y) \qquad (3)$$

EVALUATION

For the case of two orthogonal projections with their projection matrix A, the FBP or the BPF with matrix A of P(x) and P(y) of the exponentiated minimum variance result leads naturally to the density p(x, y) > 0 as the product of its marginal densities of Eqn (1), without using a cost function.

The structure of A works similarly for more than two projections. Practical implementations may prefer to use FBP, especially when the number of projections is large relative to the number of image elements.

Application in several numerical settings shows that the FBP methods works well, even when the number of projections is small and the raw projections contain significant noise. In that case, however, modest iteration and a simple regularization operation in the untransformed data domain is needed.

DISCUSSION

The formulation of Eqn (3) represents FBP or BPF in the logarithmic domain. Sample reconstructions with this concept and regularization confirm rapid convergence for any number of projections.

CONCLUSION

A logarithmic transformation of the projection data is shown to match a formulation in the probability space. Due to this transformation rapid convergence of the non-linear image reconstruction problem is demonstrated.

PATENTS

US: 8,660,328, 8,660,330; PCT: EP2310840, EP2593906; CN: ZL 200980123522.8; JP: 5543448;

Schematic of Kalman Filter Structure Applied to Noisy Iterative CT Reconstruction

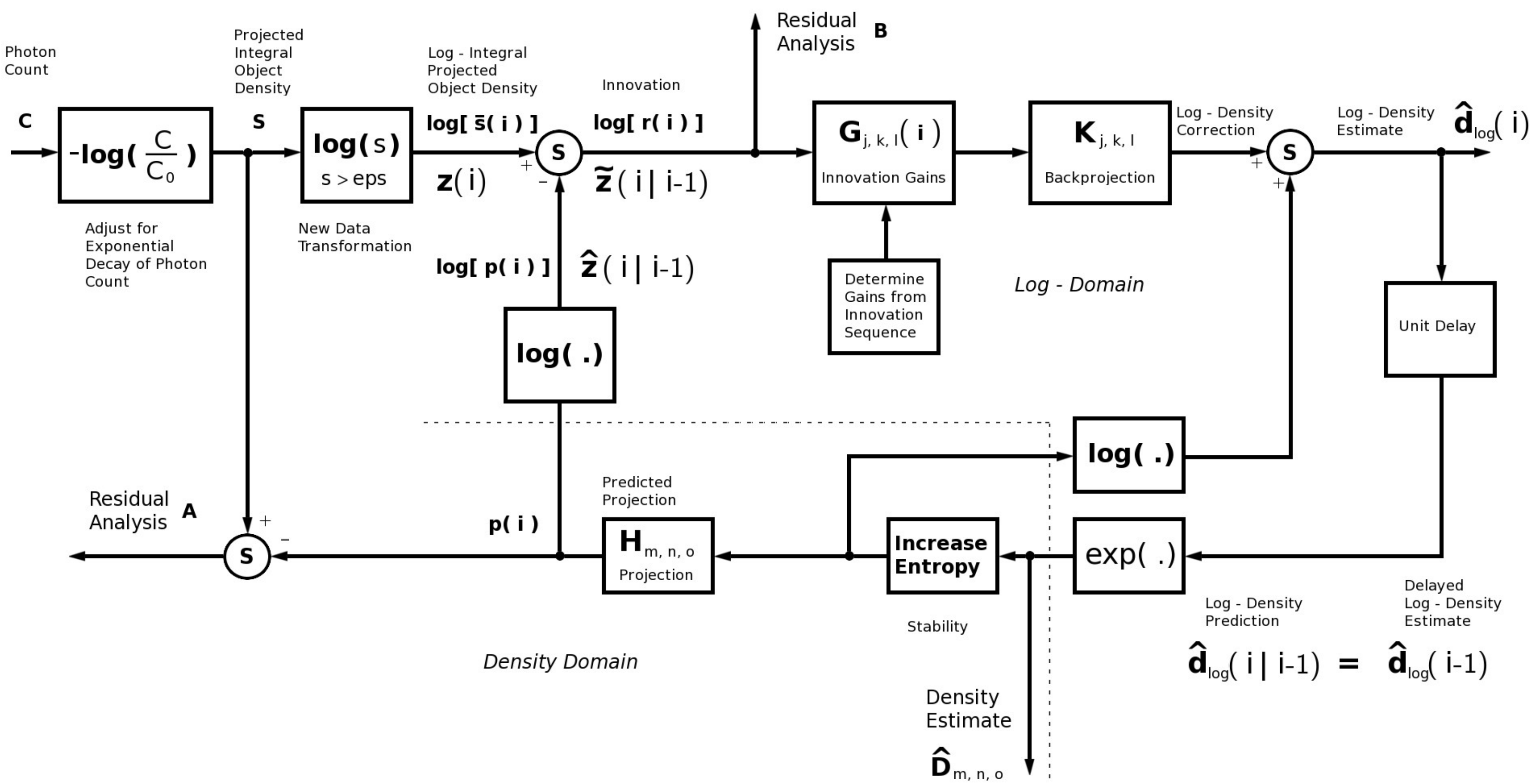

Figure 1 : Detailed information flow in HECTOR. The estimation of the innovation gain **G** cannot be pursued with the Kalman filter equations due to the size of the matrices involved. Considering the orthogonality of the components of the innovation process, however, the gain matrix **G** must be near diagonal. These diagonal elements g of **G** are then estimated from criteria of the randomness of the innovation process in the sequence of iterations. This approach leads to rapid convergence of the density estimate **D**.