Computed Tomographic (CT) Reconstruction of the Average Propagator from Diffusion Weighted MR Data

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1. Introduction

K- and q-space MRI (1) is a classical method for measuring the average propagator, *P*(*r*), in each voxel of an imaging volume using a 3-D Inverse Fourier Transform (IFT) of the diffusion weighted imaging (DWI) signal, *E*(*q*). *P*(*r*) provides key architectural and microstructural features in tissues without employing a parametric model of the diffusion process. The bottleneck in measuring *P*(*r*) *in vivo*, however, is the large number of DWIs required. Our long-term goal is to measure *P*(*r*) in its entirety in each voxel of an imaging volume using about 256 DWIs.

2. Objective

In order to obtain *P(r)*, providing the key architectural and microstructural features, we 1) reformulate the estimation of *P*(*r*) from DWI signals as a computed tomography (CT) reconstruction problem, 2) employ *a priori* information about the properties of DWIs and *P*(*r*) to constrain this reconstruction, and 3) use this reconstruction framework to optimize the experimental design.

3. Theory – Scan Time

The 3-D IFT relationship between *P(r)* and *E(q)* in each voxel is given below,

$$P(r) = \iiint E(q) e^{2\pi i q \cdot r} dq^3 \qquad \text{Eq. (1)}$$

where the vector *r* is the net displacement of spin-labeled molecules, *E(q)* is the complex-valued DW MR signal, and the vector Q specifies the strength and direction of the diffusion sensitized gradient. Brute-force, direct inversion of Eq.(1), using 3D IFFT, as in Diffusion Spectrum Imaging (DSI) (2), is too costly in scan time, in principle, requiring at least 8192 (=16 x [16x32]) DWIs to determine P(r) with high fidelity.

4. Theory – Efficient Approach

The form of Eq. (1) suggests that *P(r)* is a 3-D density, and its planar projections are isomorphic to *E(q)* slices via the Fourier slice theorem. In principle, we can then use CT reconstruction methods to estimate *P(r)* from oblique *E(q)* slices. The difference here is these CT reconstructions are performed within a single anatomical voxel within an imaging volume.

To reduce the number of DWI samples required and increase the accuracy of the tomographic reconstruction of *P(r)* we apply physical constraints, such as: (i) *P(r)* ≥ 0; (ii) *P(r) ≥ P(γ r)* ∀ *γ ≥ 1*; (iii) *E(0) ≈ E(δq) + α δqT D δq* for sufficiently small *|δq|*, where *D* is the diffusion tensor obtained from DTI and *α > 0*. Constraint (i) significantly reduces the space of possible solutions for *P(r)*; constraint (ii) requires that *P(r)* declines monotonically; constraint (iii) reduces the number of *E(q)* required at small *|δq|*.

High-Efficiency CT (HECT) (3), a new CT reconstruction method based on an approximation to Bayes Maximum a posteriori (MAP) estimation (4), is used to calculate *P(r)* from *E(q)* data obtained on different oblique planes in q-space.

Currently, HECT can accommodate constraint (i) above. Moreover, these plane projections may be noisy, the number of projections may be small (i.e., the inverse problem may be ill-posed), and arbitrary projection directions may be used.

Numerical Approach to Two Major Obstacles in Constrained CT:

1. The Hessian matrix (second derivative of objective function) is huge (here it is only 8192 x 8192 but often it is > 10^8 x 10^8).
2. The image estimate must be obtained through non-linear data manipulations.

Bayesian Setup for HECT:

With prior state statistics for the 3-D image density X

$X_0 = E[X]$, and

$P_0 = E[(X - X_0)(X - X_0)^t]$

And projection noise statistics

$E[V] = 0$, and

$R_V = E[V V^t]$, and

$E[VX^t] = 0$

Consider at first only problem #1 with the observation model $Y = H X + V$

The minimum variance estimator for the parameter X is given by the Kalman filter

$$\hat{X} = X_0 + P_0 H^t R_{YY}^{-1} (Y - H X_0)$$

$$R_{YY}^{-1} = (H P_0 H^t + R_V)^{-1}$$ non-trivial inversion possibly > 10^8 x 10^8

5. Results

We implemented several simulations of hindered and restricted diffusion using synthetic NMR "phantoms" to test the accuracy, precision, robustness, noise immunity and speed of this CT reconstruction framework of P(r). The original DWI data sets were generated with 8192 E(q) data points having 16 oblique planes passing through q=0, each with 16x32 points, whose slopes coincide with the faces of a quasi-regular icosidodecahedron. The diffusion phantom shown in Fig 1a, was generated from a 3-compartment Gaussian displacement distribution. It was found that 2048 E(q) sub-samples were sufficient to produce a 3D reconstruction of P(r) with ~1% RMS error using only four plane projections (with q-planes arranged in a tetrahedral pattern). Fig 1b. and c. show iso-probability contours of the estimated P(r) with a threshold set at 4% of the peak value, P(0), without and with 4% RMS Rician noise, respectively. Note: currently, the HECT algorithm requires at least 32 pixels in each slice dimension. On a desktop PC, each complete P(r) reconstruction takes approximately 1 second.

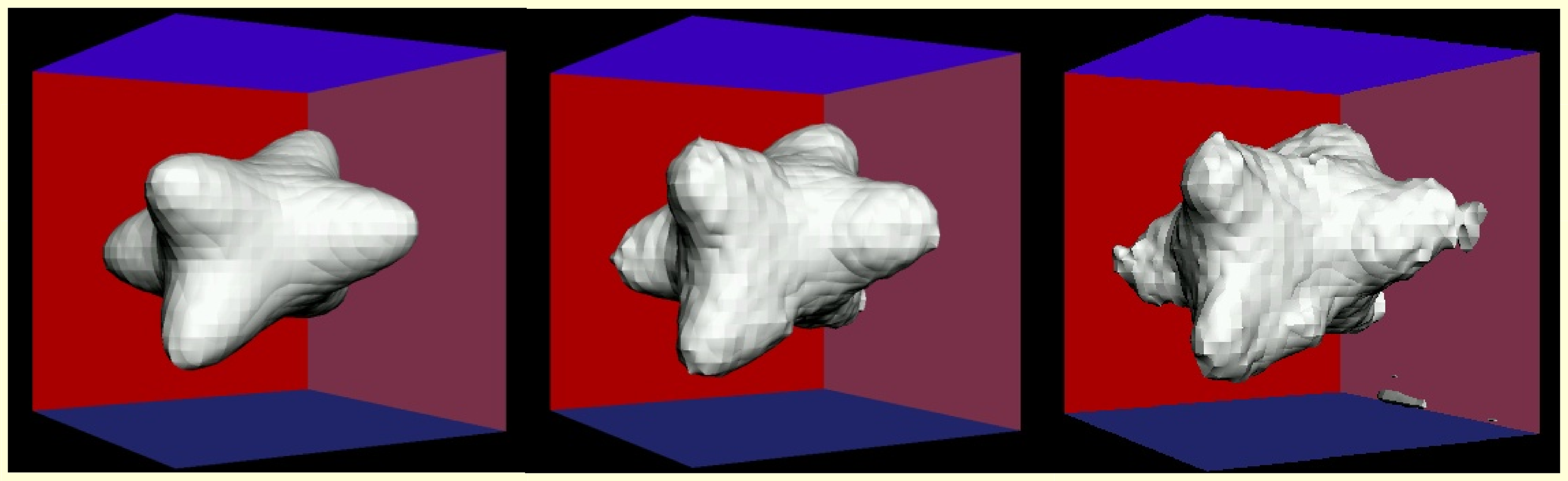

Figure 1: (a) Diffusion phantom iso-density contour at 4% of *P(0)*; (b) Four-view HECT without noise; (c) Four-view HECT with RMS Rician noise at 4% of *E(0)*.

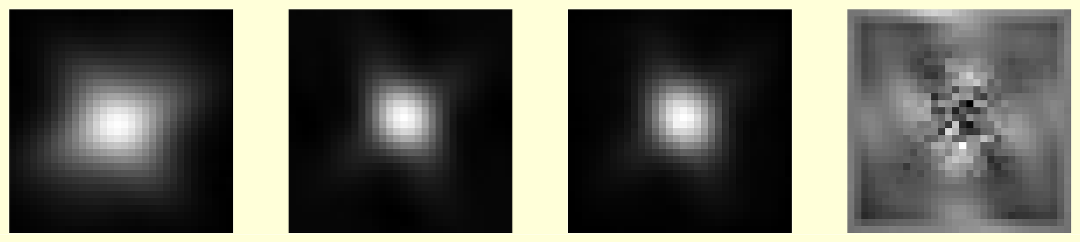

Figure 2: (a) sample slice of ***E(q)*** w/o noise; (b) corresponding projection ***P'(r')***; (c) predicted estimate $P'_e(r')$; (d) residual error ***R(r')*** with a RMS value below 1% of ***P'(0)***.

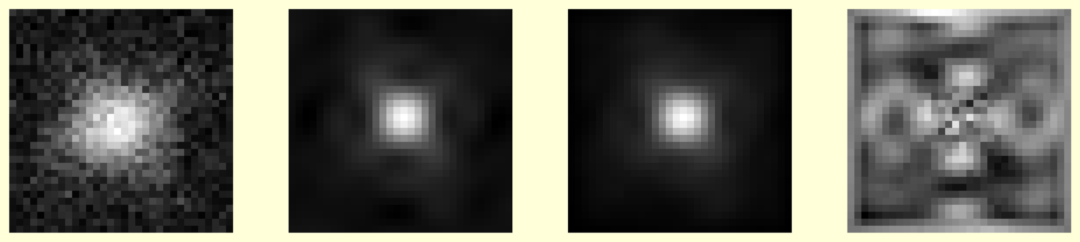

Figure 3: (a) sample slice of ***E(q)*** with 4% of ***E(0)*** RMS noise; (b) corresponding ***P'(r')***; (c) predicted estimate $P'_e(r')$; (d) residual error ***R(r')***.

Example data sets of *E(q)* slices and their transformation to projections *P'(r')* are shown in figure 2a&b w/o noise and with noise in figure 3a&b. Following reconstruction with HECT estimated projections $P'_e(r')$ are shown in figures 2c & 3c. They are compared with the original *P'(r')* and are displayed as residuals *R(r')* in figures 2d & 3d.

Although the RMS residual errors are in the one-percent range of *P'(0)* and may be regarded as small, the presence of systematic patterns implies imperfections in the present processing technique. These defects are due to known limitations of the present implementation. We anticipate to have these limitations overcome in the near future as this will allow more efficient information extraction from the DWI data.

6. Conclusions and Future Directions

Preliminary results for the HECT reconstruction of *P(r)* are encouraging. Algorithmic improvements should result in significant speedup and a four-fold reduction in the number of DWIs required from 2048 to 512 at half resolution. Further reductions in the DWI data required are expected through experimental design optimization, and by applying additional constraints and a priori information. Significant computational speedup is also possible using faster processors with multiple threads.

In summary, a new practical approach to the efficient estimation of the average propagator *P(r)* from diffusion weighted MR data is achieved with HECT. HECT is a statistically and numerically efficient new estimation technique that solves three key issues of present tomographic reconstruction - *speed, accuracy, and arbitrary number and directions of projections.*

7. References

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