Reduced CT X-Ray Dose, Improved Density Resolution, and CS MRI
High Efficiency CT with Optimized Recursions - HECTOR

Wolfram R. Jarisch, Ph.D.

Abstract

The need for accurate and fast reconstruction algorithms is presently seen as the main challenge facing the tomographic industry. Even with modern low-dose systems, CT scans often require a large number of time consuming projections in medical settings as well as non-destructive evaluations (NDE). The major shortcomings of present algorithms in CT technology are (i) inefficient data extraction and inefficient use of available information resulting in limited material densities resolution, image noise and artifacts; (ii) requirement for a large number of exposures which increases patient exposure to potentially harmful radiation in medical settings, and reduces speed of data acquisition and x-ray tube life; and (iii) slow numerical convergence, due to high computational requirements especially in cases when highest image quality is desired.

To overcome these algorithmic shortcomings, a new method, High Efficiency CT with Optimized Recursions (HECTOR) provides a significantly advanced approach to quantum efficiency and numerical efficiency for CT. As summarized in its two issued US patents, HECTOR efficiently solves the statistical inversion problem of tomographic reconstructions and provides an objective quality control of each reconstruction through its residual analysis. Extracting all information in a verifiable manner (the residuals are white) intrinsically leads to better image quality and hence improved diagnostic quality. The present goal is to embed circular and helical cone-beam algorithms in the HECTOR structure, as needed in modern CT scanners, and in Compressed Sensing MRI. This approach provides improved estimation of material densities in a wide range of settings.

Theoretical Underpinnings of HECTOR

Work by S. Wood & M. Morf (1981) [1] showed a minimum variance / Bayesian setup for tomography. Their work demonstrated the occurrence of significant numerical problems when handling or inverting very large matrices. In modern tomography, their approach would entail a multiple of $10^{20}$ operations – far out of reach of computers for the foreseeable future. They also showed that certain matrices had specific “reasonable” structures, but it was not clear whether that information could be used in a numerical cost-saving approach. Their approach has been largely abandoned and superseded as a result of these and other perceived insurmountable problems.

A number of quite sophisticated methods have been developed to improve upon Radon’s (1917) [2] original concept. Subsequent practical implementation of CT in the early 1970’s relied on Filtered Back Projections (FBP) requiring high radiation levels. Soon thereafter, iterative techniques were recognized as a new approach to reduce artifacts and radiation levels. Today, some of the leading competing techniques are known with acronyms like ART (algebraic relaxation technique), SIRT (statistical iterative reconstruction technique), ASIR (adaptive statistical iterative reconstruction), and MBIR (model based iterative reconstruction).

One of the most notable of these techniques is the iterative set-up by DeMan (2006) [3] for GE, a closed-loop iterative structure in the presence of limited measurement quality and quantity. Although DeMan's setup is quite appealing from a pragmatic perspective, it is very inefficient when only a small number of projections is available.

Others, like Bian et al. (2013) [4] and Fessler (1990; 2010) [5] developed a different approach to perform reconstructions from a small number of projections. However, their approach requires massive iterative and parallel computation to overcome their non-linear processing of the data. Their approach is inefficient since it does not take advantage of simplifying techniques, in particular, the orthogonalization of density updates or the estimation of the Kalman-like gain matrix, as discussed in Sage & Melsa (1971) [6], and S. Wood & M. Morf (1981) [1].

The present PI has developed a more advanced reconstruction technique termed High Efficiency CT with Optimized Recursions (HECTOR) that leads to a rapid convergence of iterations and overcomes DeMan's rate of convergence limitations. HECTOR's underpinnings derive from a Bayesian Kalman filtering technique designed to solve a nonlinear problem by applying an orthogonalization transformation. This orthogonalization optimization is applied in addition to DeMan's -log x-ray measurement transformation. As a result, HECTOR
handles the data extraction more efficiently and requires a reduced number of measurements (presently implemented on a single-threaded x86_64 CPU). HECTOR reconstructions have been demonstrated with as few as four projections (for diffusion tensor MRI) to as much as hundreds of projections for double-tilt transmission electron microscopy (TEM) tomography.

To demonstrate the concept, consider an extremely simple case: assume two orthogonal projections without measurement noise. DeMan's algorithm would require many iterations to correctly estimate a single point in an empty space. It may also not be numerically stable. HECTOR provides the correct solution without any iterations: the estimate is simply the maximum entropy solution, the product of the marginal densities, e.g.:

\[ p( x, y, z) = p( x, z) \cdot p( y, z) \]  

or, equivalently [ assume for simplicity \( p( . ) > 0 \) ]

\[ \log[ p( x, y, z)] = \log[ p( x, z)] + \log[ p( y, z)] \]  

where \( p( u, v) \) represents the projection information after the -log transformation of the x-ray measurements. The additional log-transformation in (2) produces additive operations similar to filtered back-projections techniques. The basis of the HECTOR algorithm [7,8] combines the concept of this log-transformation with a “locally linearized” non-linear Kalman-like filter setup.

The present PI recognized several opportunities for numerical simplification of the estimation problem while adding non-linear constraints to the 3D density estimation setup. In particular, the non-linear setup follows an iterative approach that may be modeled by a sequential Kalman filter process. Although the Kalman filtering / Bayesian setup leads to a non-linear process, the present approach applies them as the basis for formulating a small-signal linearized innovation process with efficient near orthogonal updates.

Effective linearization is achieved through multigrid processing, resulting in small changes, as follows: first, assume that the whole space is a single voxel of known density (the projection of that voxel produces the same total integral projection in any direction); second, divide up the space into smaller subspaces, e.g., 2x2x2 sub-voxels; third, perform linearized operations and a few numerically inexpensive iterations that allow for the efficient estimation of the correction terms from the previous one-voxel density; and finally, repeat this concept of sub-dividing the voxel-space until the desired resolution is reached (for details see patents [7,8]).

Since the estimation is closely related to filtered back-projections, the solution is closely related to minimum variance estimation. The final objective is to obtain an estimate of the appropriate relaxation matrix without resorting to the above-noted multiple of \( 10^{20} \) operations (recall Wood & Morf; 1981). Rather than through explicit computation [1], this is achieved through a “back-door” based on the optimality properties of the relaxation gain matrix, whereby the residual components must not oscillate as the estimation process progresses [8]. For sample applications see www.tomography3d.com.

Value of High Efficiency CT with Optimized Recursions (HECTOR)

The high statistical and numerical efficiency of HECTOR adds significant value to a broad range of imaging applications in the health care sector and industrial fields, as well as for the military and homeland security:

- The HECTOR algorithm can be embedded as a software upgrade in X-ray equipment to substantially increase the CT performance and dose reduction in medical applications, NDE, luggage inspection, and other settings
- In compressed sensing MRI settings, HECTOR’s efficiency leads to: increased patient load which translates into cost-and-time savings and / or increased image resolution
- Reduced measurement time in Diffusion Tensor Imaging (DTI)
- More broadly, HECTOR can improve the imaging of objects especially in time-sensitive environments where minimizing the impact of measurement while achieving high-quality imaging is of the essence.

The improvements in these imaging applications are enabled by HECTOR’s patented technical innovations:

- Numerical efficiency leading to short computing times as a result of performing iterations at progressive resolution, beginning with a single voxel;
- Progressive resolution leading to small signal innovations with better linearization of the non-linear part of the estimation problem and correction of the effect of imperfect filter designs;
- Logarithmic transformation of the error signal leading to a near linearization of the closed loop feedback (depending on the need for the smoothing operation). Note that the s/p division is equivalent to the difference of logarithms: \( \log \left( \frac{s}{p} \right) = \log(s) - \log(p) \). In this way, the second log-transform keeps loop variable values approximately in a single (logarithmic) domain;
- Utilization of an approximate inverse impulse response filter, corresponding to Radon’s inversions, resulting in a near orthogonal set of output innovations (comparable to traditional FBP reconstruction);
- Maintaining the update loop near linear (due to the near logarithmic-domain variables in the feedback loop) and thus stabilizing the coefficients of the Kalman-like (m-dimensional) gain matrix throughout the iterations;
- Avoiding the common concept of explicit computation of the Kalman / Wiener filter loop gains through matrix or spectral computations (the loop is not exactly linear); utilizing instead the sequence of historical innovation variables (e.g. observing their oscillations and rate of changes);
- Maintaining the Kalman-like gain-matrix stable due to the small scale innovations and near-linear feedback loop;
- Applying the Kalman-like gain matrix to help correct deficiencies of projection matrix approximations;
- Providing a separate filter as needed to correct projection updates prior to the log-density update (accounting for the distribution of projections such as for the missing wedge in TEM or low projection count);
- Determination of the coefficients of a new, optimizing Kalman-like, gain matrix (sparse, diagonally dominant) from the history of innovations. This matrix applies throughout the progressive resolutions (each with their own iterations) with modest change;
- Enabling visual and statistically quantifiable quality control of the high statistical efficiency either from the difference image matrices \((s – p)\) or the sensitive error ratio matrix;
- Providing numerical stability for ill-conditioned settings, such as low projection counts in TEM CT and in vascular imaging as a result of the mild smoothing operation;
- Achieving high numerical efficiency by minimizing the number of computationally expensive final high-resolution iterations, if any.
- For examples of algorithm performance see www.tomography3d.com.
Figure 1a: Exemplary implementation of High Efficiency CT with Optimized Recursions (HECTOR). Building on small scale linearization the algorithm operates in three domains as follows:

1. Conversion of the raw x-ray measurements to obtain the integral material density representation (traditional);

2. Systematic **progressive resolution** of all density representations results in small amplitude changes, **effective linearization of changes** (new), and reduction in the **initial computational burden** of iterations (new);

3. **Ratio transformation** of the observed material density representation with its prediction (new);

4. **Log-transformation** of the projection density error (new). The small scale **linearized orthogonalization** (new) of process innovations, related to Radon’s inversion with independent projections, proceeds with an approximate **Kalman-like Matrix filter gain** (new) that, combined with FBP (and possible corrections), provides near optimal log-density updates.

Starting out with a single voxel as the entire space gradually refines the grain of the image resolution high numerical efficiency is achieved. In a 3-D space, the initial coarse iterations can be carried out with a relatively low computational effort, as compared to the greater effort focused on the final few iterations.

Note the structural simplicity of this approach that overcomes the need to apply various iterative nonlinear “hill climbing” approaches. As a result of the innovations described above, the HECTOR algorithm leads to rapid and accurate convergence. It uses only a small fraction of time of other nonlinear algorithms but only modestly more than filtered back projection. The approach applies for example to x-ray CT, TEM, and without the conversion of intensity, to compressed sensing MRI. **Existing FBP algorithms may be embedded in HECTOR.**
Figure 1b: Detailed information flow in HECTOR. The estimation of the innovation gain $G$ cannot be pursued with the Kalman filter equations due to the size of the matrices involved. Considering the orthogonality of the components of the innovation process, however, the gain matrix $G$ must be near diagonal. These diagonal elements $g$ of $G$ are then estimated from criteria of the randomness of the innovation process in the sequence of iterations. This approach leads to rapid convergence of the density estimate $\hat{D}$. 
The accuracy of the reconstruction can be observed objectively by an analysis of the data residuals: the HECTOR algorithm has been demonstrated to leave data residuals as white noise which provides a quantifiable quality control for efficient information extraction.

Many examples of the performance of the HECTOR algorithm can be seen at www.tomography3D.com.

Transmission electron microscopy (TEM) and artificial noise challenge data were provided by the NIBIB of the National Institutes of Health for comparing their FBP and SIRT with Cyber Technology's HECT.

Figure 2: Slice-comparison of 180 view double tilt TEM reconstruction-from-projections.

Figure 3a: Sample of 48 projection “noisy cube” challenge (provided by the NIBIB of NIH).

Figure 3b: Quality control: data residuals for NIH’s nonlinear SIRT algorithm with 15 iterations (left) vs. the HECT algorithm for the “noisy cube challenge” with a numerical effort near 1 iteration. Lack of cube structure in the HECT residuals implies effective information extraction. Note: HECT is a precursor of HECTOR.
Figures A4 & A5: GE's algorithm structure for comparison with HECTOR's (see Fig. 1): from Bernard DeMan et al. pending patent US20060072801 A1, Pub. Date Apr. 6, 2006

Method and System for Iterative Image Reconstruction
Assignee: General Electric Company
Related to the structures shown of Figs. 4 & 5, Yin et al. (US patent 7,215,732 B2, assignee GE) approach the optimization of a positive density constraint of the detector photon count via an exponential transformation of the related detector output variable (their Equt. 12 uses the exponential constraint \( C(s) = \| y - H \exp(-s) \|_w^2 \); see their US patent claim #11). Yin et al. note “… the optimization problem may be solved using any standard non-linear optimization method such as for example, gradient search. …”.

The concept of the exponential transformation for constraining a variable is also found in HECTOR. There, however, it is applied to voxel densities instead of photon counts. Using equts. (1) and (2) for maximum entropy estimation of the joint density from its marginal densities leads naturally to this exponential transformation.

The Oct. 29 2013 Patent US 8,571,287 B2 by DeMan et al., Assignee GE, uses a pragmatic approach with varying resolution computations, regional relaxation factors, and optimization approaches like Krylov subspace optimization and iterative error descent, as summarized in their Fig. 4. Note that no second layer of a logarithmic transformation as in HECTOR is provided.

Figure A6: Overall processing schematic of the nonlinear tomographic reconstruction in the recent patent by DeMan et al. (their FIG. 2), System and Method for Iterative Image Reconstruction.